Vector coherent states of non-compact orthosymplectic Lie supergroups

1990 J. Phys. A: Math. Gen. 23 L43

(http://iopscience.iop.org/0305-4470/23/2/001)

J. Phys. A: Math. Gen. **23** (1990) L43–L48. Printed in the UK

LETTER TO THE EDITOR

Vector coherent states of non-compact orthosymplectic Lie supergroups

C Quesne†

Physique Nucléaire Théorique et Physique Mathématique CP229, Université Libre de Bruxelles, Bd du Triomphe, B1050 Bruxelles, Belgium

Received 26 October 1989

Abstract. Vector coherent states are defined for the generic lowest-weight ladder irreducible representations of the non-compact orthosymplectic Lie supergroups $\mathrm{OSp}(2M/2N, \mathbb{R})$ and $\mathrm{OSp}(2M+1/2N, \mathbb{R})$. Their simultaneous use with K-matrix theory is shown to provide a powerful tool for the study of such representations. This is illustrated with the detailed example of $\mathrm{OSp}(1/2N, \mathbb{R})$.

Over the past decade, Lie supergroups and Lie superalgebras have come to play an important role in theoretical physics. They indeed underlie all supersymmetric theories such as superstring and supergravity theories. In this context, one is often interested in lowest-weight ladder irreducible representations (irreps) of non-compact supergroups (Bars and Günaydin 1983). Such is the case, for instance, for the four-dimensional $N=8$ anti-de Sitter supergroup $\mathrm{OSp}(8/4, \mathbb{R})$, which was applied to the S^7 compactification of the eleven-dimensional supergravity (Günaydin and Warner 1986).

Non-compact orthosymplectic supergroups $\mathrm{OSP}(P/2N, \mathbb{R})$, where $P=2M$ or $2M+1$, also make their appearance at a less fundamental level as groups of canonical transformations for mixed systems of bosons and fermions (de Crombrugghe and Rittenberg 1983, Balantekin *et al* 1988, 1989). Many applications were recently reported in various fields, ranging from disordered electron systems (Wegner 1983) to stochastic quantum mechanics (Verbaarschot *et al* 1985) and nuclear spectroscopy (Schmitt *et al* 1988, 1989).

In a parallel development, the coherent states (CS) of quantum optics (Glauber 1963a, b), which provide a natural link between classical and quantum mechanics and are related to the path integral formalism, were extended to arbitrary Lie groups (Perelomov 1972, 1977, Gilmore 1972, 1974). Generalised CS for a Lie group G are defined by acting with some irrep of G on a fixed vector carrying a one-dimensional irrep of a subgroup H. Later on, they were further extended to deal with finite-dimensional vector representations of H (Deenen and Quesne 1984, Rowe 1984, Rowe *et al* 1985, Quesne 1986). These are the so-called vector coherent states (VCS), whose combination with K-matrix theory provides a powerful tool in Lie group and Lie algebra representation theory (Rowe 1984, Hecht 1987, Rowe *et al* 1988).

† Directeur de recherches FNRS.

In spite of their potential usefulness, up to now relatively little attention has been paid to the CS and VCS for supergroups (Bars and Günaydin 1983, Günaydin 1988, Balantekin *et al* 1988, 1989, Le Blanc and Rowe 1989). Standard (generalised) CS for the most degenerate lowest-weight unitary irreps of $\mathrm{OSp}(1/2N, \mathbb{R})$ and $\mathrm{OSp}(2/2N, \mathbb{R})$ were recently introduced and analysed in detail (Balantekin *et al* 1988, 1989). However, VCS for the generic lowest-weight irreps of $\mathrm{OSp}(P/2N, \mathbb{R})$ have not been considered so far.

The basic role played by Lie algebra gradings in the construction of Lie groups VCS is well known (Rowe *et al* 1988, Quesne 1989b). On the other hand, it has been shown that such gradings can be extended to Lie superalgebras to give a unified construction of both types of mathematical structures (Bars and Günaydin 1979). Hence, one can guess that the VCS construction, introduced for Lie groups, works equally well for Lie supergroups. This point was recently illustrated with the examples of $\mathrm{Gl}(M+N)$ and $\mathrm{Gl}(M/N)$ (Le Blanc and Rowe 1989).

It is the purpose of the present letter and of a more detailed forthcoming paper to study the $\mathrm{OSp}(P/2N, \mathbb{R})$ VCS. Here we shall review the definition of the corresponding superalgebras and of their lowest-weight irreps, introduce the VCS of the orthosymplectic supergroups, and finally illustrate their usefulness in Lie supergroup and Lie superalgebra representation theory by treating in detail the case of $\mathrm{OSp}(1/2N, \mathbb{R})$.

The non-compact $\mathrm{osp}(2M/2N, \mathbb{R})$ superalgebra is spanned by the operators

$$\Lambda_{AB} = (-1)^{\eta_A\eta_B}\Lambda_{BA} = (\Lambda_{-B,-A})^\dagger \qquad A, B = \pm 1, \ldots, \pm(M+N) \tag{1}$$

satisfying the supercommutation relations

$$[\Lambda_{AB}, \Lambda_{CD}\} = g_{CB}\Lambda_{AD} - (-1)^{(\eta_A+\eta_B)(\eta_C+\eta_D)}g_{AD}\Lambda_{CB} + (-1)^{\eta_A\eta_B}(g_{CA}\Lambda_{BD} - (-1)^{(\eta_A+\eta_B)(\eta_C+\eta_D)}g_{BD}\Lambda_{CA}) \tag{2}$$

where

$$\eta_A = \begin{cases} 1 & \text{if } A = \pm 1, \ldots, \pm M \qquad (3a) \\ 0 & \text{if } A = \pm(M+1), \ldots, \pm(M+N) \qquad (3b) \end{cases}$$

$$g_{AB} = \delta_{A,-B}\varepsilon_A \tag{4}$$

and

$$\varepsilon_A = \begin{cases} 1 & \text{if } A = \pm 1, \ldots, \pm M \qquad (5a) \\ A/|A| & \text{if } A, B = \pm(M+1), \ldots, \pm(M+N). \qquad (5b) \end{cases}$$

The star adjoint relation given in (1) implies that we shall restrict ourselves to star representations†. Only $\mathrm{osp}(2/2N, \mathbb{R})$ has, in addition, grade star representations (Scheunert *et al* 1977), which might be treated in a similar way.

A basis for the even part of $\mathrm{osp}(2M/2N, \mathbb{R})$ consists of the $\mathrm{so}(2M)$ and $\mathrm{sp}(2N, \mathbb{R})$ generators, respectively defined by

$$A^\dagger_{ab} = \Lambda_{ab} \qquad A^{ab} = \Lambda_{-b,-a} \qquad C_a{}^b = \Lambda_{a,-b} \qquad a, b = 1, \ldots, M \tag{6}$$

and

$$D^\dagger_{ij} = \Lambda_{M+i,M+j} \qquad D^{ij} = \Lambda_{-M-i,-M-j} \qquad E_i{}^j = \Lambda_{M+i,-M-j} \qquad i, j = 1, \ldots, N \tag{7}$$

† For the even generators, the star adjoint condition is determined by the Hermitian adjoint relationships valid for $\mathrm{so}(P)$ and $\mathrm{sp}(2N, \mathbb{R})$. For the odd generators, it is fixed up to an overall sign (Scheunert *et al* 1977); the sign chosen here corresponds to a realisation in a super Fock space.

where $C_a{}^b$ and $E_i{}^j$ span u(M) and u(N) subalgebras. Its odd part has basis elements

$$G^\dagger_{ai} = \Lambda_{a,M+i} \qquad G^{ai} = \Lambda_{-a,-M-i} \qquad H_i{}^a = \Lambda_{-a,M+i}$$
$$J_a{}^i = \Lambda_{a,-M-i} \qquad a = 1, \ldots, M \qquad i = 1, \ldots, N. \tag{8}$$

Equations (1)-(5) also define the osp($2M+1/2N, \mathbb{R}$) superalgebra provided the range of indices $A, B, \ldots$ is extended to $0, \pm 1, \ldots, \pm(M+N)$ in (1), (2) and (4), and to $0, \pm 1, \ldots, \pm M$ in (3a) and (5a). Then, to the operators of (6), one has to add

$$B^\dagger_a = \Lambda_{a0} \qquad B^a = \Lambda_{0,-a} \qquad a = 1, \ldots, M \tag{9}$$

to obtain the so($2M+1$) generators. In the same way, the odd generators (8) have to be supplemented with the operators

$$F^\dagger_i = \Lambda_{0,M+i} \qquad F^i = \Lambda_{0,-M-i} \qquad i = 1, \ldots, N. \tag{10}$$

In most physical applications, one is interested in the decomposition of the osp($P/2N, \mathbb{R}$) irreps into irreps of the even subalgebra so(P)⊕sp($2N, \mathbb{R}$). In the solution to this problem given below, a central role is played by the maximal compact even subalgebra so(P)⊕u(N) of osp($P/2N, \mathbb{R}$)†. We shall therefore consider the subalgebra chain

$$\mathrm{osp}(P/2N, \mathbb{R}) \supset \mathrm{so}(P) \oplus \mathrm{sp}(2N, \mathbb{R}) \supset \mathrm{so}(P) \oplus \mathrm{u}(N). \tag{11}$$

The osp($2M/2N, \mathbb{R}$)[osp($2M+1/2N, \mathbb{R}$)] generators can be realised in a super Fock space $\mathcal{F}$ as bilinear operators in $Mn[(M+1)n]$ pairs of fermion creation and annihilation operators and Nn pairs of boson creation and annihilation operators (Günaydin 1988, Günaydin and Hyun 1988). The lowest-weight ladder osp($P/2N, \mathbb{R}$) irreps realised in $\mathcal{F}$ can be characterised by the lowest-weight so(P)⊕sp($2N, \mathbb{R}$) irrep $[\Xi] \oplus \langle \Omega \rangle$ (or equivalently by the lowest-weight so(P)⊕u(N) irrep $[\Xi] \oplus \{\Omega\}$) contained in their carrier space, and they will therefore be denoted by $[\Xi\Omega\rangle$. Here $[\Xi]$, $\langle \Omega \rangle$, and $\{\Omega\}$ are shorthand notation for $[\Xi_1 \Xi_2 \ldots \Xi_M]$, $\langle \Omega_1 \Omega_2 \ldots \Omega_N \rangle$, and $\{\Omega_1 \Omega_2 \ldots \Omega_N\}$, respectively. Provided n is large enough, generic irreps $[\Xi] \oplus \{\Omega\}$ of so(P)⊕u(N) (thence generic irreps $[\Xi\Omega\rangle$ of osp($P/2N, \mathbb{R}$)) can be obtained in this realisation. Note that the irreps considered by Balantekin *et al* (1988, 1989) correspond to the case where $\Xi_1 = \ldots = \Xi_M$ and $\Omega_1 = \ldots = \Omega_N$.

Let $|[\Xi]\{\Omega\}\alpha\rangle$ denote basis states of the lowest-weight so(P)⊕u(N) irrep $[\Xi] \oplus \{\Omega\}$. By definition, they are annihilated by the osp($P/2N, \mathbb{R}$) lowering generators G^{ai}, $J_a{}^i$, D^{ij} (and F^i whenever $P = 2M+1$). Application of the raising generators $G^\dagger_{ai}$, $H_i{}^a$, $D^\dagger_{ij}$ (and $F^\dagger_i$ whenever $P = 2M+1$) to such states then generates the whole carrier space of the osp($P/2N, \mathbb{R}$) irrep $[\Xi\Omega\rangle$.

The construction of OSp($P/2N, \mathbb{R}$) VCS is based on the Kantor decomposition or five-dimensional $\mathbb{Z}$-graded structure (Bars and Günaydin 1979) of osp($P/2N, \mathbb{R}$) with respect to its maximal compact even subalgebra so(P)⊕u(N):

$$\mathrm{osp}(P/2N, \mathbb{R}) = g_{-2} \oplus g_{-1} \oplus g_0 \oplus g_{+1} \oplus g_{+2} \tag{12}$$

where

$$g_{-2} = \mathrm{span}\{D^{ij}\} \qquad g_{-1} = \mathrm{span}\{F^i, G^{ai}, J_a{}^i\} \qquad g_0 = \mathrm{so}(P) \oplus \mathrm{u}(N)$$
$$g_{+1} = \mathrm{span}\{F^\dagger_i, G^\dagger_{ai}, H_i{}^a\} \qquad g_{+2} = \mathrm{span}\{D^\dagger_{ij}\} \tag{13}$$

† This contrasts with the maximal compact subsuperalgebra u(M/N), consdered by Günaydin (1988) and Günaydin and Hyun (1988).

and F^i and $F_i^\dagger$ have to be dropped whenever $P=2M$. The generator giving the grading is $\mathbb{N}=E_i^{\ i}$, where there is a summation over repeated upper and lower indices. The intermediate algebra in (11) has a Jordan decomposition or three-dimensional $\mathbb{Z}$-graded structure (Bars and Günaydin 1979) with respect to the same subalgebra:

$$\mathrm{so}(P)\oplus\mathrm{sp}(2N,\mathbb{R})=g_{-2}\oplus g_0\oplus g_{+2}. \tag{14}$$

An arbitrary vector Z belonging to the complex extension $g_{-2}\oplus g_{-1}$ can be expanded as

$$Z=\tfrac{1}{2}z_{ij}D^{ij}+\theta_iF^i+\sigma_{ai}G^{ai}+\tau_i^{\ a}J_a^{\ i} \tag{15}$$

where $z_{ij}=z_{ji}$, $i,j=1,\ldots,N$, are complex (commuting) variables, and θ_i, σ_{ai}, $\tau_i^{\ a}$, $a=1,\ldots,M$, $i=1,\ldots,N$, are complex (anticommuting) Grassmann variables. Note that, in (15) and in the equations to follow, the variables θ_i have to be dropped whenever $P=2M$. The variables z_{ij}, θ_i, σ_{ai}, $\tau_i^{\ a}$ parametrise the complex extension of the supercoset space $\mathrm{OSp}(P/2N,\mathbb{R})/[\mathrm{SO}(P)\otimes\mathrm{U}(N)]$.

The $\mathrm{OSp}(P/2N,\mathbb{R})$ VCS are then defined by

$$|z,\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\tau};\alpha\rangle=\exp(Z^\dagger)|[\Xi]\{\Omega\}\alpha\rangle. \tag{16}$$

The VCS representation of an arbitrary state Ψ, belonging to the irrep $[\Xi\Omega\rangle$ carrier space, and of an $\mathrm{osp}(P/2N,\mathbb{R})$ generator X are given by

$$\Psi(z,\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\tau})=\langle z,\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\tau}|\Psi\rangle=\sum_\alpha|[\Xi]\{\Omega\}\alpha\rangle\langle[\Xi]\{\Omega\}\alpha|\exp(Z)|\Psi\rangle \tag{17}$$

and

$$\Gamma(X)\Psi(z,\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\tau})=\sum_\alpha|[\Xi]\{\Omega\}\alpha\rangle\langle[\Xi]\{\Omega\}\alpha|\exp(Z)X|\Psi\rangle \tag{18}$$

respectively. The function $\Psi(z,\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\tau})$ is a holomorphic function in the complex variables z_{ij}, a polynomial in the Grassmann variables θ_i, σ_{ai}, $\tau_i^{\ a}$, and it takes vector values in the lowest-weight $\mathrm{so}(P)\oplus\mathrm{u}(N)$ subspace. The operator $\Gamma(X)$ can be expressed as a differential operator on $\Psi(z,\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\tau})$, depending in addition on the representation $\mathbb{A}_{ab}^\dagger$, $\mathbb{A}^{ab}$, $\mathbb{B}_a^\dagger$, $\mathbb{B}^a$, $\mathbb{C}_a^{\ b}$, and $\mathbb{E}_i^{\ j}$ of $\mathrm{so}(P)\oplus\mathrm{u}(N)$ carried by the lowest-weight invariant subspace.

The integral form of the VCS identity resolution being both difficult to obtain and cumbersome to use, we shall not try to determine it as was done by Balantekin *et al* (1988, 1989) for the standard CS of $\mathrm{OSp}(1/2N,\mathbb{R})$ and $\mathrm{OSp}(2/2N,\mathbb{R})$. We shall, instead, adhere to the general philosophy of VCS and K-matrix combined theory, wherein the VCS scalar product is defined by specifying an orthonormal basis in the VCS representation space. For this purpose, a basis orthonormal with respect to a Bargmann–Berezin scalar product (Bargmann 1961, Berezin 1966) is first constructed. The extension of the Bargmann scalar product, used when dealing with Lie algebras, to a Bargmann–Berezin one is due to the additional presence of Grassmann variables and is connected with the replacement of boson CS (Glauber 1963a, b) by boson-fermion CS (Ohnuki and Kashiwa 1978). The orthonormal Bargmann–Berezin basis is then mapped onto an orthonormal VCS basis by means of a transformation K. In terms of the latter, one can define an $\mathrm{osp}(P/2N,\mathbb{R})$ representation

$$\gamma(X)=K^{-1}\Gamma(X)K \tag{19}$$

equivalent to the VCS representation $\Gamma(X)$ and star adjoint with respect to the Bargmann–Berezin scalar product.

To illustrate such a procedure, we shall now consider the example of the $\mathrm{osp}(1/2N, \mathbb{R})$ irreps $[\Omega\rangle$ in more detail. This case is simple because all $\mathrm{osp}(1/2N, \mathbb{R})$ irreps are known to be typical (Scheunert 1979).

The VCS are parametrised by the complex variables z_{ij}, the Grassmann variables θ_i, and the discrete index α labelling a basis $|\{\Omega\}\alpha\rangle$ of the lowest-weight $\mathrm{u}(N)$ subspace. The Baker-Campbell-Hausdorff formula leads to the following VCS expansion for the generators of $\mathrm{osp}(1/2N, \mathbb{R})$:

$$\Gamma(D^{ij}) = \nabla^{ij} \qquad \Gamma(F^i) = \partial^i + \tfrac{1}{2}\theta_j \nabla^{ji} \qquad \Gamma(E_i{}^j) = \mathbb{E}_i{}^j + z_{ik}\nabla^{kj} + \theta_i \partial^j$$
$$\Gamma(F_i^\dagger) = z_{ij}\partial^j + \theta_j(\mathbb{E}_i{}^j + \tfrac{1}{2}z_{ik}\nabla^{kj} + \tfrac{1}{2}\theta_i\partial^j) \tag{20}$$
$$\Gamma(D_{ij}^\dagger) = (z_{ik} + \tfrac{1}{2}\theta_i\theta_k)\mathbb{E}_j{}^k + (z_{jk} + \tfrac{1}{2}\theta_j\theta_k)\mathbb{E}_i{}^k + z_{ik}z_{jl}\nabla^{kl} + (\theta_i z_{jk} + \theta_j z_{ik})\partial^k$$

where $\nabla^{ij} = (1+\delta_{ij})\partial/\partial z_{ij}$, and $\partial^i = \partial/\partial\theta_i$.

An orthonormal Bargmann-Berezin basis of vector-valued functions reducing the $\mathrm{u}(N)$ subalgebra is given by

$$\langle z, \boldsymbol{\theta}|[\Omega\rangle\{1^l\dot{0}\}\langle\omega\rangle\{\nu\}\rho\{h\}\chi\rangle = [P^{\{\nu\}}(z)[Q^{\{1^l\dot{0}\}}(\boldsymbol{\theta})|\{\Omega\}\rangle]^{\{\omega\}}]^{\rho\{h\}}_{\chi} \tag{21}$$

in terms of some polynomials $P^{\{\nu\}}_\beta(z)$, $Q^{\{1^l\dot{0}\}}_\gamma(\boldsymbol{\theta})$, transforming under the $\mathrm{u}(N)$ irreps $\{\nu\} = \{\nu_1\nu_2\ldots\nu_N\}$ and $\{1^l\dot{0}\}$, where ν_i are non-negative even integers and a dot over a numeral implies that this numeral is repeated as often as necessary. Here the square brackets denote $\mathrm{u}(N)$ couplings, $\{\omega\} = \{\omega_1\omega_2\ldots\omega_N\}$ and $\{h\} = \{h_1h_2\ldots h_N\}$ characterise $\mathrm{u}(N)$ irreps; β, γ, χ label $\mathrm{u}(N)$ bases, and ρ distinguishes between repeated $\{h\}$ irreps.

The transformation K, mapping the basis (21) onto an orthonormal VCS basis

$$\langle z, \boldsymbol{\theta}|\varphi([\Omega\rangle\{1^l\dot{0}\}\langle\omega\rangle\rho\{h\}\chi)\rangle = \langle z, \boldsymbol{\theta}|K|[\Omega\rangle\{1^l\dot{0}\}\langle\omega\rangle\{\nu\}\rho\{h\}\chi\rangle \tag{22}$$

can be chosen in such a way that $\gamma(E_i{}^j) = \Gamma(E_i{}^j)$, and that the states (22) reduce the chain (11), now reading $\mathrm{osp}(1/2N, \mathbb{R}) \supset \mathrm{sp}(2N, \mathbb{R}) \supset \mathrm{u}(N)$. Hence, the K matrix is diagonal in $\{h\}$ and independent of χ, the $\mathrm{sp}(2N, \mathbb{R})$ irreps are characterised by $\langle\omega\rangle = \langle\omega_1\omega_2\ldots\omega_N\rangle$, and the construction of an orthonormal VCS basis may be restricted to that of a basis of lowest-weight $\mathrm{u}(N)$-irrep states $\langle z, \boldsymbol{\theta}|\varphi([\Omega\rangle\{1^l\dot{0}\}\langle\omega\rangle\{\dot{0}\}\{\omega\}\chi)\rangle$. In the subspace spanned by these states, the K matrix is block diagonal in $\langle\omega\rangle$, and the diagonal blocks $\mathcal{K}(\{\omega\})$ are one dimensional.

By a method analogous to that used by Rowe *et al* (1988), it can be shown that $\mathcal{K}(\{\omega\})^2$ satisfies the recursion relation

$$\mathcal{K}(\{\omega'\})^2 = (\Omega_i - \mathrm{i} + 1)\left(\prod_{k=1}^{l}(\Omega_i + \Omega_{p_k} - p_k - \mathrm{i} + 2)(\Omega_i + \Omega_{p_k} - p_k - \mathrm{i} + 1)^{-1}\right)\mathcal{K}(\{\omega\})^2 \tag{23}$$

where $\{\omega\} = \{\Omega + \Delta^{(l)}(p_1, \ldots, p_l)\}$, $\{\omega'\} = \{\Omega + \Delta^{(l+1)}(p_1, \ldots, p_m, \mathrm{i}, p_{m+1}, \ldots, p_l)\}$, and $\Delta^{(l)}(p_1, \ldots, p_l)$ denotes a row vector of dimension N with vanishing entries everywhere except for the components $p_1, \ldots, p_l$, which have value unity. By starting from $\mathcal{K}(\{\Omega\}) = 1$, and choosing the positive square root in (23), all the submatices $\mathcal{K}(\{\omega\})$ can be easily determined; thence a VCS basis of lowest-weight $\mathrm{u}(N)$-irrep states can be built. Since the construction of orthonormal $\mathrm{sp}(2N, \mathbb{R}) \supset \mathrm{u}(N)$ bases was extensively studied (Rowe 1984, Deenen and Quesne 1985, Hecht 1987), the whole VCS basis can in fact be determined. As a by-product of K-matrix theory, we obtain the following branching rule for the decomposition of the $\mathrm{osp}(1/2N, \mathbb{R})$ irrep $[\Omega\rangle$ into $\mathrm{sp}(2N, \mathbb{R})$

irreps $\langle\omega\rangle$:

$$[\Omega]\downarrow \sum_{\omega_1=\Omega_1}^{\Omega_1+1} \sum_{\omega_2=\Omega_2}^{\min(\Omega_2+1,\,\omega_1)} \cdots \sum_{\omega_N=\Omega_N}^{\min(\Omega_N+1,\,\omega_{N-1})} \oplus\langle\omega\rangle. \tag{24}$$

By using the Bargmann–Berezin basis (21) and the γ representation (19), it is straightforward to determine the matrix elements of the odd generators $F_i^\dagger$, F^i between two lowest-weight u(N)-irrep basis states. They can indeed be expressed in terms of a ratio of $\mathcal{K}$ submatrices and of the corresponding matrix elements of θ_i and ∂^i between two Bargmann–Berezin states. This enables one to rederive equation (19) of Quesne (1989a) by using the full power of u(N) tensor calculus instead of a tedious and difficult to generalise raising operator technique.

In a forthcoming publication, we plan to extend to OSp($P/2N,\mathbb{R}$) the analysis carried out for OSp($1/2N,\mathbb{R}$) in the present letter. The implementation of K-matrix theory will then make use of so(P)$\oplus$u(N) tensor calculus. For small values of P and (or) N, it will be possible to derive in a very simple way some important fundamental results such as branching rules and explicit matrix representations.

References

Balantekin A B, Schmitt H A and Barrett B R 1988 *J. Math. Phys.* **29** 1634
Balantekin A B, Schmitt H A and Halse P 1989 *J. Math. Phys.* **30** 274
Bargmann V 1961 *Commun. Pure Appl. Math.* **14** 187
Bars I and Günaydin M 1979 *J. Math. Phys.* **20** 1977
—— 1983 *Commun. Math. Phys.* **91** 31
Berezin F A 1966 *The Method of Second Quantization* (New York: Academic)
de Crombrugghe M and Rittenberg V 1983 *Ann. Phys., NY* **151** 99
Deenen J and Quesne C 1984 *J. Math. Phys.* **25** 2354
—— 1985 *J. Math. Phys.* **26** 2705
Gilmore R 1972 *Ann. Phys., NY* **74** 391
—— 1974 *Rev. Mex. Fis.* **23** 143
Glauber R J 1963a *Phys. Rev.* **130** 2529
—— 1963b *Phys. Rev.* **131** 2766
Günaydin M 1988 *J. Math. Phys.* **29** 1275
Günaydin M and Hyun S J 1988 *J. Math. Phys.* **29** 2367
Günaydin M and Warner N P 1986 *Nucl. Phys.* B **272** 99
Hecht K T 1987 *The Vector Coherent State Method and its Application to Problems of Higher Symmetries* (*Lecture Notes in Physics* **290**) (Berlin: Springer)
Le Blanc R and Rowe D J 1989 *J. Math. Phys.* **30** 1415
Ohnuki Y and Kashiwa T 1978 *Prog. Theor. Phys.* **60** 548
Perelomov A M 1972 *Commun. Math. Phys.* **26** 222
—— 1977 *Usp. Fiz. Nauk* **123** 23 (*Sov. Phys.-Usp.* **20** 703)
Quesne C 1986 *J. Math. Phys.* **27** 428, 869
—— 1989a *J. Phys. A: Math. Gen.* **22** L355
—— 1989b Vector coherent state theory of the semidirect sum Lie algebras wsp($2N,\mathbb{R}$) *J. Phys. A: Math. Gen.* (submitted)
Rowe D J 1984 *J. Math. Phys.* **25** 2662
Rowe D J, Le Blanc R and Hecht K T 1988 *J. Math. Phys.* **29** 287
Rowe D J, Rosensteel G and Gilmore R 1985 *J. Math. Phys.* **26** 2787
Scheunert M 1979 *The Theory of Lie Superalgebras* (*Lecture Notes in Mathematics* **716**) (Berlin: Springer)
Scheunert M, Nahm W and Rittenberg V 1977 *J. Math. Phys.* **18** 146
Schmitt H A, Halse P, Balantekin A B and Barrett B R 1989 *Phys. Rev.* C **39** 2419
Schmitt H A, Halse P, Barrett B R and Balantekin A B 1988 *Phys. Lett.* **210B** 1
Verbaarschot J J M, Weidenmüller H A and Zirnbauer M R 1985 *Phys. Rep.* **129** 367
Wegner F 1983 *Z. Phys.* B **49** 297